Series 2 for FEM

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1 Maxwell equation

$$\nabla \times (\frac{1}{\mu_r} \mathrm{S} \nabla \times \boldsymbol{E}) - k_0^2 \epsilon_r \mathrm{S}^{-1} \boldsymbol{E} = -jk_0 Z_0 \boldsymbol{J} \tag{1}$$

Here, μ_r and ϵ_r denote the relative permeability and permittivity of the medium; while for the PML region, they represent the property of the medium adjacent to the PML region; k_0 and z_0 denote the vacuum wavenumber and impedance respectively. The matrix S is defined as follows, for the non-PML region,

$$\mathrm{S} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix};$$

for the PML region,

$$\mathrm{S} = \begin{bmatrix} s_x/(s_y s_z) & 0 & 0 \\ 0 & s_y/(s_x s_z) & 0 \\ 0 & 0 & s_z/(s_x s_y) \end{bmatrix}.$$

Taking the x-component as an example,

$$\begin{aligned} s_x &= k_x + \frac{\sigma_x}{j\omega_0 \epsilon} = k_x + \frac{\sigma_x z_0}{j} \frac{1}{k_0 \epsilon_r}, \\ k_x &= 1 + (k_{x,max} - 1)(x/d)^m, \\ \sigma_x &= \sigma_{x,max}(x/d)^m. \end{aligned}$$

It is necessary to point out that s_x is a function of wavenumber k_0. The second form of s_x is straightforward for us to get its derivatives with respect to k_0.