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From here all values will be automatically calculated in accordance with your input.

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***11 Degree of parallelism**

The degree of parallelism will be automatically calculated as the product of the values of three variables; (1), (2), and (3) as shown in *4.

***12 Effective parallelization ratio α(%)**

The effective parallelization ratio is calculated differently depending on the case of strong or weak scaling. The effective parallelization ratio of both strong scaling and weak scaling are calculated automatically in this Excel sheet based on the different formulae. Please utilize these values to set the target of the project (cf. *16 Target degree of parallelism).

The details of the equations are as follows;

(a) In case of strong scaling

The ratio of the elapsed time of the parallelized part to the total elapsed time and that of the serial part to the total are defined as α and (1-α), respectively. This α will be deduced by employing the Amdahl's law. With this evaluation method, the effective parallelization ratio including the overheads due to parallelization can be obtained. Specifically, the effective parallelization ratio can be obtained through the following calculations.

Measure elapsed times, T_m and T_n at two points of parallelism, m and n (n>=2m). Here, we define T_1 as an elapsed time with the parallelism of 1. Following Amdahl's law, the ratio of speed-up with the parallelism of m and n can be defined as:

$$S_n = \frac{T_1}{T_n} = \frac{1}{1 - \alpha + \frac{\alpha}{n}} \qquad (1)$$

$$S_m = \frac{T_1}{T_m} = \frac{1}{1 - \alpha + \frac{\alpha}{m}} \qquad (2)$$

By eliminating T_1 from equations (1) and (2), the effective parallelization ratio of strong scaling α^s is expressed as:

$$\alpha^S \equiv \alpha = \frac{T_m - T_n}{\left(1 - \frac{1}{n}\right) T_m - \left(1 - \frac{1}{m}\right) T_n} \qquad (3)$$

If you enter values of T_m and T_n in the Excel Sheet for the case of strong scaling, α^s will be calculated automatically by using the equation (3) and shown in terms of %.

The calculated effective parallelization ratio might be larger than 1 (unity) or negative, if the elapsed time is measured with small m. Please choose proper conditions to avoid such situations.

(b) In case of weak scaling

The elapsed time of the parallelized part is assumed to be constant. In addition, some part of the serial part is also assumed to be constant. The elapsed time of these parts is assumed to be α in total, and the residual serial part whose elapsed time is dependent on the size of data is assumed to increase in proportion to the size of data. From these assumptions, the relation below holds:

$$T_n : T_m = \alpha + (1-\alpha)n : \alpha + (1-\alpha)m \qquad (4)$$

The effective parallelization ratio of weak scaling α^w can be solved as follows:

$$\alpha^W \equiv \alpha = \frac{nT_m - mT_n}{(1-m)T_n - (1-n)T_m} \qquad (5)$$

In case of weak scaling measurements of T_m and T_n, α^w will be automatically calculated with the equation (5).

***13 Parallel Efficiency (E_n)**

The parallel efficiency 'E_n' for a given degree of parallelism 'n' indicates how much the program is efficiently accelerated by parallel processing. 'E_n' is given by the following formulae. Although their derivation processes are different depending on strong and weak scaling, derived formulae are the same.

(a) In case of strong scaling

In case of strong scaling, 'E_n^S' is calculated automatically with the following formula, and is displayed in percentage in the sheet:

$$E_n^S = \frac{S_n}{n} = \frac{1}{(1-\alpha^S)n + \alpha^S} \qquad (6)$$

where α is the effective parallelization ratio, given by the formula (3) with the speed-up ratio 'S_n', given by Amdahl's law.

For example, if n=100, and 'S_n' = 20, 50, and 100 times, 'E_n^S' are 20, 50, and 100%, respectively. If α exceeds 1, 'E_n^S' becomes larger than 100% or less than 0%, and as a result, Amdahl's law is no longer applicable. Then, please try to avoid those situations by properly choosing conditions of performance measurement.

(b) In case of weak scaling

In case of weak scaling, the amount of computation for a given degree of parallelism 'n' is n times larger than the case with n=1. Ideally, the elapsed time should be the same regardless of whether the software is running in serial or parallel processing. In this case, 'E_n^W' is supposed to be 100%. 'E_n^W' is 50% if the elapsed time in parallel processing is twice as long as that of the serial processing, and 20% if 5 times. Such 'E_n^W' is expressed as a simple ratio of the elapsed time in serial processing to that in parallel processing, and is calculated with a simple formula as follows:

$$E_n^W = \frac{T_1}{T_n} = \frac{1}{(1-\alpha^W)n + \alpha^W} \qquad (7)$$

Here, (7) is derived from (4).

Although the ways to derive the effective parallelization ratio α are different depending on strong and weak scaling cases, the formulation of the 'E_n' is the same in the end in both cases. 'E_n^W' is calculated automatically with the formula (6) or (7), and is displayed in percentage.

***14 Guideline for degree of parallelism**

The degree of parallelism 'n' which gives 'E_n' = 50% is calculated automatically with the following formula (9).

$$E_n = \frac{1}{(1-\alpha)n + \alpha} = \frac{1}{2} \qquad (8)$$

$$\mathrm{n} = \frac{2-\alpha}{1-\alpha} \qquad (9)$$

This value is just a guideline. You can set a larger degree of parallelism as a target, especially if you are planning to speed up your program after starting your project, or if the reduction of the elapsed time is more important than improvement of the parallel efficiency.

***15 Target degree of parallelism**

The target degree of parallelism will be automatically calculated as the product of the values of three variables; (1), (2), and (3) as shown in *10

***16 Estimated parallel efficiency (E_x)**

The estimated parallel efficiency (E_x) is calculated automatically with the formula (6) or (7) by assigning the target degree of parallelism (x) to 'n', and the result is shown in %. Please use this value to check whether the value of (x) is appropriate or not.