

My model is a bead packing. I simulate each bead contact with a spring. This is what the spring network looks like overlapped with the beads packing.

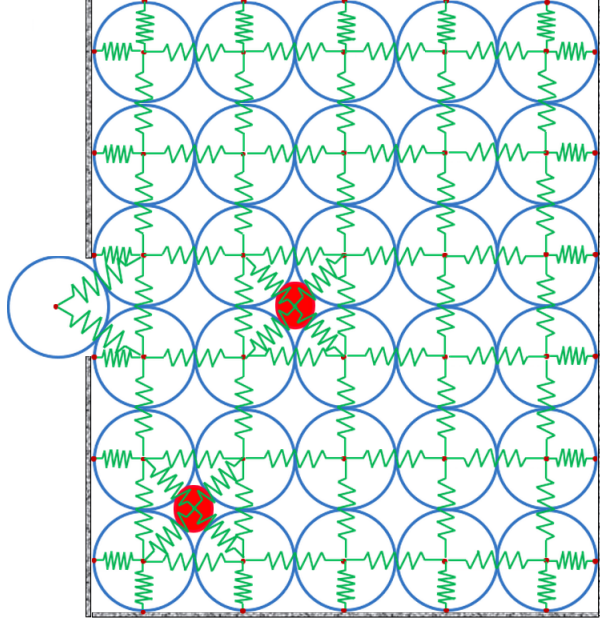


Figure 1: Discrete Element Model for the bead packing

Equilibrium equations, each node has two degrees of freedom.

$$\mathbf{R}^{(n)}(\mathbf{u}^{(n)}) = \mathbf{M}\ddot{\mathbf{u}}^{(n)} + \mathbf{K}\mathbf{u}^{(n)} - \mathbf{F}_{ext}^{(n)} = 0 \quad (1)$$

The scalar force in each spring is just

$$F_{spring} = k_{sp}\delta \quad (2)$$

where k_{sp} is the spring coefficient and δ is the relative displacements between the nodes.

Each spring contributes with the following internal force vector:

$$\mathbf{F}_{spring} = k_{sp}\delta \cdot (\mathbf{t}, -\mathbf{t}) \quad (3)$$

The vector \mathbf{t} is the unit vector that connects the springs' nodes.

$$\mathbf{t} = \frac{(\mathbf{X}_i - \mathbf{X}_j)}{|\mathbf{X}_i - \mathbf{X}_j|} \quad (4)$$

Each spring contributes with the following constitutive matrix (assuming small deformations):

$$\mathbf{K}_{spring} = k_{sp}\delta \cdot (\mathbf{t}, -\mathbf{t}) \otimes (\mathbf{t}, -\mathbf{t}) \quad (5)$$

The mass matrix is just a diagonal matrix, one entry per degree of freedom.