Simple Analytical Model of ALOHA Network

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This is a greatly simplified model of a radio network. Each node in the network generates packets conforming to a poisson distribution (given rate R_{gen}). These packets are routed along fixed paths to a sink. When two packets are send at the same time, they are dropped. No channel sensing or retransmission takes place. Each active link (i.e. each radio link that will eventually transmit data) is modeled as a vertex. Between those vertices there exist two kinds of relations:

$R_I(l_a, l_s)$ implies that whenever the l_s tries to send at the same time than l_a, the packet will collide and is not received properly.

$R_F(l_a, l_s)$ implies that each packet that is send via l_s link and does not collide, adds up to the amount of packets send via l_a.

$$I_i = \{j \mid R_I(l_i, l_j)\}, \tag{1}$$

$$F_i = \{j \mid R_F(l_i, l_j)\} \tag{2}$$

The overall rate of accessing a radio link i is

$$R_{access,i} = R_{gen,i} + \sum_{j \in F_i} R_{success,j}. \tag{3}$$

The probability that no other link interfering with i accesses the channel at a given point in time is

$$P_{free,i} = \prod_{j \in I_i} (1 - R_{access,j}). \tag{4}$$

Finally, the overall rate of successful transmissions over link i is

$$R_{success,i} = R_{access,i} \cdot P_{free,i}. \tag{5}$$

This builds up the equation system

$$f_{a,i}(x) = 0 = g_i + \sum_{j \in F_i} x_{s,j} - x_{a,i}, \tag{6}$$

$$f_{s,i}(x) = 0 = x_{a,i} \left(\prod_{j \in I_i} (1 - x_{a,j}) \right) - x_{s,i}. \tag{7}$$

The non-zero first-order partial derivatives for building up the Jacobian are

$$\frac{\partial f_{a,i}}{\partial x_{a,i}} = -1, \tag{8}$$

$$\frac{\partial f_{a,i}}{\partial x_{s,j}} = 1 \qquad \text{for } j \in F_i, \tag{9}$$

$$\frac{\partial f_{s,i}}{\partial x_{a,i}} = \prod_{j \in I_i} (1 - x_{a,j}), \tag{10}$$

$$\frac{\partial f_{s,i}}{\partial x_{a,j}} = -x_{a,i} \cdot \prod_{k \in I_i \wedge k \neq j} (1 - x_{a,k}) \qquad \text{for } j \in I_i, \tag{11}$$

$$\frac{\partial f_{s,i}}{\partial x_{s,i}} = -1. \tag{12}$$