4 Matrix form

$$\begin{bmatrix} \mathbf{H} & \mathbf{0} & -\mathbf{N} \\ \mathbf{0} & \mathbf{0} & -\mathbf{A}^{\mathrm{T}} \\ \mathbf{M} & -\mathbf{A} & \mathbf{F} \end{bmatrix} \begin{Bmatrix} \delta\mathbf{r} \\ \delta\mathbf{q} \\ \delta\mathbf{s} \end{Bmatrix} = \begin{bmatrix} -\mathbf{R}_{\rho}{}_{n+1}^{i} \\ -\mathbf{R}_{\sigma}{}_{n+1}^{i} \\ -\mathbf{R}_{\varepsilon}{}_{n+1}^{i} \end{bmatrix} \tag{26}$$

$$\mathbf{H} = \frac{1}{\Delta t}\int_{\Omega} (\mathbf{P}_{\rho})^{\mathrm{T}}\mathbf{P}_{\rho}\mathrm{d}\Omega - \int_{\Omega} (\mathbf{P}_{\rho})^{\mathrm{T}}\frac{\partial\mathscr{R}}{\partial\rho}(\sigma_{n+1}^{i},\rho_{n+1}^{i})\mathbf{P}_{\rho}\mathrm{d}\Omega \tag{27}$$

$m > n$ guarantee uniqueness of the solution

$$\mathbf{N} = \int_{\Omega} (\mathbf{P}_{\rho})^{\mathrm{T}}\frac{\partial\mathscr{R}}{\partial\sigma}(\sigma_{n+1}^{i},\rho_{n+1}^{i})\mathbf{S}\mathbf{v}\mathrm{d}\Omega \tag{28}$$

$$\mathbf{A} = \int_{\Gamma} (\mathbf{N}\mathbf{S}_{\mathbf{v}})^{\mathrm{T}}\mathbf{u}_{\Gamma}\mathrm{d}\Gamma \tag{29}$$

$$\mathbf{M} = \int_{\Omega} (\mathbf{S}_{\mathbf{v}})^{\mathrm{T}}\frac{\partial\varepsilon}{\partial\rho}(\sigma_{n+1}^{i},\rho_{n+1}^{i})\mathbf{P}_{\rho}\mathrm{d}\Omega \tag{30}$$

$$\mathbf{F} = \int_{\Omega} (\mathbf{S}_{\mathbf{v}})^{\mathrm{T}}\frac{\partial\varepsilon}{\partial\sigma}(\sigma_{n+1}^{i},\rho_{n+1}^{i})\mathbf{S}_{\mathbf{v}}\mathrm{d}\Omega \tag{31}$$

5 Partial solution

$$\delta\mathbf{r} = \mathbf{H}^{-1}(-\mathbf{R}_{\rho}{}_{n+1}^{i} + \mathbf{N}\delta\mathbf{s}) \tag{32}$$

$$\delta\mathbf{s} = (\mathbf{F}+\mathbf{M}\mathbf{H}^{-1}\mathbf{N})^{-1}(-\mathbf{R}_{\varepsilon}{}_{n+1}^{i} + \mathbf{M}\mathbf{H}^{-1}\mathbf{R}_{\rho}{}_{n+1}^{i} + \mathbf{A}\delta\mathbf{q}) \tag{33}$$

$$-\mathbf{A}^{\mathrm{T}}(\mathbf{F}+\mathbf{M}\mathbf{H}^{-1}\mathbf{N})^{-1}\mathbf{A}\delta\mathbf{q} = -\mathbf{R}_{\sigma}{}_{n+1}^{i} - \mathbf{A}^{\mathrm{T}}(\mathbf{F}+\mathbf{M}\mathbf{H}^{-1}\mathbf{N})^{-1}(-\mathbf{R}_{\varepsilon}{}_{n+1}^{i} + \mathbf{H}^{-1}\mathbf{R}_{\rho}{}_{n+1}^{i}) \tag{34}$$