SAMPLE FILE FOR THE PROBLEM

LULU LIU

1. Equations for the incompressible two-phase flow . For each time step, we need to solve a nonlinear system (with two unknowns—pressure p and saturation s)

$$F(p,s) = \begin{bmatrix} G(p,s) \\ H(p,s) \end{bmatrix} = 0 \tag{1.1}$$

and the initial values (p_0, s_0) are given.

2. What I want to do. Instead solving the original system (1.1), we want to solve

$$\hat{F}(p,s) = \begin{bmatrix} \hat{G}(p,s) \\ \hat{H}(p,s) \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix} = 0 \tag{2.1}$$

where u and v are obtained by solving

$$G(p-u,s) = 0, \qquad H(p, s-v) = 0. \tag{2.2}$$

The Jacobian of (2.1) is

$$\hat{F}'(p,s) = \begin{bmatrix} G_p^{-1} & \\ & H_s^{-1} \end{bmatrix} \begin{bmatrix} G_p & G_s \\ H_p & H_s \end{bmatrix} \tag{2.3}$$