Table 1: Estimated values for turbulent channel and pipe flow.

Re_b	5000	10000	36700	50000	100000	367000	500000
Re_t	298.68	549.68	1725.9	2265.7	4169.7	13092	17187
δy_1	.003348	.001819	.000579	.000441	.000240	.000076	.000058
u_τ	.059735	.054968	.047027	.045314	.041697	.035674	.034374
τ_w	.003568	.003022	.002211	.002053	.001739	.001272	.001181
Re_t	298.29	547.07	1706.6	2236.9	4102.5	12798	16774
δy_1	.003352	.001828	.000586	.000447	.000244	.000078	.000060
u_τ	.059659	.054707	.046501	.044738	.041025	.034871	.033549
τ_w	.003559	.002993	.002162	.002002	.001683	.001216	.001126
Re_t	342.87	628.82	1961.6	2571.1	4715.5	14710	19281
δy_1	.002917	.001590	.000510	.000389	.000212	.000068	.000052
u_τ	.068573	.062882	.053449	.051423	.047155	.040081	.038562
τ_w	.004702	.003954	.002857	.002644	.002224	.001606	.001487
τ_2	.004710	.003960	.002861	.002648	.002227	.001608	.001489
Re_2	343.37	629.75	1964.5	2574.9	4722.5	14732	19309
$\frac{\partial p}{\partial x}$	.018809	.015817	.011427	.010577	.008894	.006426	.005948

1 Turbulent Channel Flow Parameters

Table 1 provides a distribution of values for $Re_\tau := u_\tau h/\nu$ and derived quantities estimated for turbulent channel flow as a function of the bulk Reynolds number, $Re_b := Uh/\nu$, where U is the mean flow velocity and h is the channel half-height.[1] The quantities are computed using the approximate formula

$$\begin{aligned} Re_\tau &= 0.166\, Re_b^{0.88} && \text{for channel flow (1st group)} \\ Re_\tau &= 0.173\, Re_b^{0.875} && \text{for channel flow (2nd group)} \\ Re_\tau &= 0.123\, Re_D^{0.875} && \text{for pipe flow} \end{aligned}$$

along with the relations

$$\begin{aligned} u_\tau = \sqrt{\tau_w/\rho} = \frac{\nu}{h} Re_\tau, &\qquad u^+ = u/u_\tau, \qquad y^+ = u_\tau y/\nu, \\ y^+(y=h) = Re_\tau, &\quad y(y^+=1) = 1\nu/u_\tau, \quad \tau_w = \rho u_\tau^2, \\ \frac{\partial p}{\partial x} = 2\tau_w/R &\qquad . \end{aligned}$$

δy_1 is the value of y such that y^+=1 and is therefore an estimate of the near-wall grid spacing in the wall-normal direction. The last equation relates the wall shear stress to the mean pressure drop for flow in a pipe of radius R. (This is the quantity FFX used by Nek5000 to force the flow in periodic pipe flow.)

Example: Rudman and Blackburn[2] performed LES of pipe flow at Re=UD/ν=36,700 using a spectral/SEM code with a Smagorinsky model and van Driest damping for the eddy viscosity.

[1] These quantities were computed using the matlab .m file `loglaw_scales`.

[2] *Large Eddy Simulation of Turbulent Pipe Flow*, 2nd Int. Conf. on CFD in the Minerals and Process Industries, CSIRO, Melbourne, Australia, December, 1999.

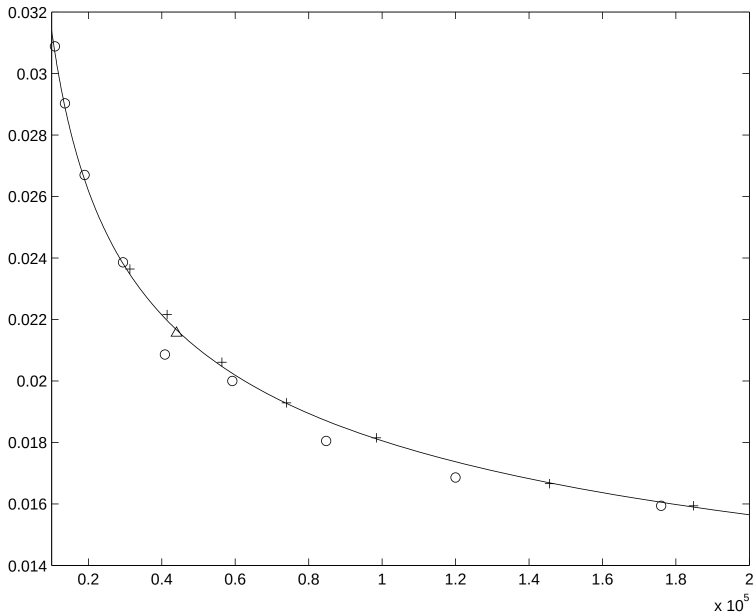

Figure 1: Friction factor λ from Oregon ($\circ$) and Princeton ($+$) experiments [?] and DNS ($\triangle$) [?] for $10,000 \leq Re_D \leq 200,000$, along with fit given by 1.

Their first grid point in the wall normal direction was at r^+=1, with $r^+ := u_\tau(R-r)/\nu$. In the axial direction they targeted $\Delta z^+ \approx$50–100 and chose $\Delta z = L_z/192 \approx 0.0327$, with $L_z = 2\pi D$. In the spanwise direction, they targeted $(R\Delta\theta)^+ \approx$15–40, but had an effective value of 45 for N=10. Their total resolution was 1.2 million mesh points. The azimuthal spacing is thus 2–3 times as dense as the axial spacing, and the wall-normal spacing at the wall is roughly 15–45 times denser than the azimuthal spacing. The authors investigate the effects of the Smagorinsky constant by considering C_s=0.065 and 1.0.

For pipe flow at Re=100,000 one would need Δr=.00024, and Δz=.024, and $R\Delta\theta$

2 Recent DNS and Experimental Data

Wu and Moin [?] computed turbulent flow at $Re_D = 44,000$ using a second-order code with 630 million grid points. They report $R^+ := u_\tau R/\nu = 1142$, which corresponds to $u_\tau = 2284/44000 = .05191$ and $-\frac{dp}{dx} = 4u_\tau^2 = .0107782$. We note that, in nondimensional units, the friction factor λ is twice the (negative) mean pressure drop.

McKeon et al. [?] report a fit of friction factor vs. Reynolds number based on data from recent pipe flow experiments in Oregon and Princeton. Their fit is of the form

$$\frac{1}{\sqrt{\lambda}} = a\ln(Re\sqrt{\lambda}) \; - \; b, \tag{1}$$

with constants $a = 1.930$ and $b = 0.537$ fitting over the Reynolds number range $31,000 \leq Re \leq 35,000,000$. For data in the range of interest to our present simulations, $10,000 \leq Re \leq 200,000$, we find $a = 2.045$ $b = 1.0$ to give a superior fit. Unfortunately, this range is centered in the ambiguous overlap regime where the Oregon experiment is near the upper range of its accessible Reynolds numbers and the Princeton experiment is near the lower range. Interestingly, the Wu and Moin data is in between this range of values.