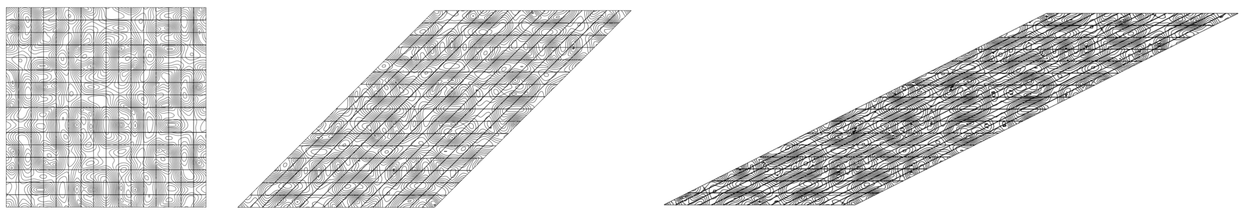
Navier-Stokes Eigenfunctions - Sheared Grid

This example extends the Navier-Stokes eigenfunctions [1] example (Nek5000/short_tests/eddy) to characterize the impact of element shear on the accuracy of the spectral element method.

Consider the shear in x as a function of y: $y = s$; $x = r + s\eta$ where η is a constant. Choosing η to be an integer ensures that original periodicity is preserved for the top and bottom rows of elements with no change to the element connectivity. (This shear can thus be introduced in `usrdat2`.) Figure 1 shows the vorticity on the 16×16 grid used in the Eddy example, along with solutions for $\eta = 1$ and $\eta = 2$.



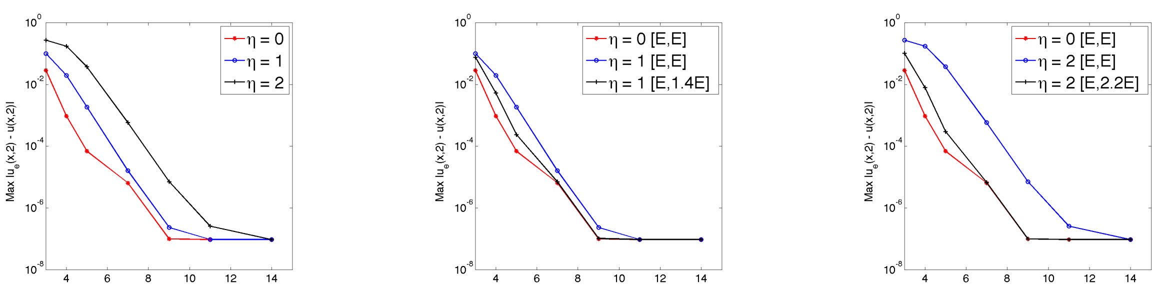
$\eta = 0$ $\eta = 1$ $\eta = 2$

Figure 1: 16x16 grids

Shearing is a rather benign mesh deformation in that the Jacobian and metrics $(\frac{\partial x_i}{\partial r_j})$ remain constant. As η is increased, however, spectral element edges that were originally vertical are stretched by a factor $\sqrt{1+\eta^2}$, and the resolution is effectively decreased. The impact of this resolution reduction is evident in Fig. 2(a), which shows that, while the error increases with η, spectral convergence is retained. (The conditions are the same as in the Eddy example: $\nu = .05$, $\mathbf{u}_0 = (16, 5)$, $P_N - P_{N-2}$, $t_{order} = 3$, and $dt = .0001$, final time $t = 2$.)

Figures 2(b) and (c) illustrate that the original convergence behavior is recovered simply by increasing the number of elements in the y direction so that the resolution along the stretched edge is restored to the value of the original mesh. That is, the number of elements in y is increased by $\sqrt{1+\eta^2}$.

Sheared meshes do not significantly degrade the SEM accuracy other than yielding suboptimal resolution along the sheared edge. This resolution must be recovered by using more elements along the edge that is stretched.



16x16 grids $\eta = 0$ and 1 $\eta = 0$ and 2

Figure 2: Error comparison for sheared grids

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[1] O. Walsh, "Eddy solutions of the Navier-Stokes equations," *The NSE II-Theory and Numerical Methods*, J.G. Heywood, K. Masuda, R. Rautmann, and V.A. Solonikkov, eds., Springer, pp. 306–309 (1992)