I am hoping to compute some unusual quantities using Nek5000. To make a long story short, I want to solve the 3D Rayleigh-Benard problem

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} - \nu\Delta\mathbf{u} + \nabla p = -g\mathbf{k}(1-\alpha\delta_T(1+T)), \tag{0.1}$$

$$\nabla\cdot\mathbf{u} = 0, \tag{0.2}$$

$$\frac{\partial T}{\partial t} + (\mathbf{u}\cdot\nabla)T - \beta\Delta T = 0, \tag{0.3}$$

supplemented with the boundary conditions

$$\mathbf{u} = 0 \text{ at } z = 0 \text{ and } z = h, \tag{0.4}$$

$$T(0) = 1, \quad T(h) = 0, \tag{0.5}$$

$$p, \mathbf{u}, T \text{ periodic in horizontal directions } x, y, \tag{0.6}$$

in the domain

$$\Omega = \{(x,y,z) : (x,y) \in [0,L]\times[0,L], 0 \le z \le h\}. \tag{0.7}$$

Here $\mathbf{u} = (u, v, w)$.

The unusual thing is that in the process of solving (0.1)-(0.3), I also want to compute the L^2-norms of the quantities

$$\frac{\partial u}{\partial z}(x,y,h), \quad \frac{\partial u}{\partial z}(x,y,0), \quad \frac{\partial v}{\partial z}(x,y,h), \quad \frac{\partial v}{\partial z}(x,y,0)$$

as well as

$$\int_0^h \frac{\partial}{\partial x}\left[(u-\tilde{u})^2\right] dz, \quad \int_0^h \frac{\partial}{\partial y}[(v-\tilde{v})(u-\tilde{u})]dz$$

and

$$\int_0^h \frac{\partial}{\partial x}[(u-\tilde{u})(v-\tilde{v})]dz \quad \int_0^h \frac{\partial}{\partial y}\left[(v-\tilde{v})^2\right] dz\ .$$

where

$$\tilde{u} = \tilde{u}(x,y) = \frac{1}{h}\int_0^h u(x,y,z)\ dz\ ,$$

and similarly for v.

Does anyone see a reasonable way to do this using Nek5000?